Twistorial Analyticity and Three Stringy Systems of the Kerr Spinning Particle

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Abstract

The Kerr spinning particle has a remarkable analytical twistorial structure. Analyzing electromagnetic excitations of the Kerr circular string which are aligned to this structure, we obtain a simple stringy skeleton of the spinning particle which is formed by a topological coupling of the Kerr circular singular string and by an axial singular stringy system.

We show that the chiral traveling waves, related to an orientifold world-sheet of the axial stringy system, are described by the massive Dirac equation, so we argue that the axial string may play the part of a stringy carrier of wave function and play also a dominant role in the scattering processes.

A key role of the third, *complex* Kerr string is discussed. We conjecture that it may be one more alternative to the Witten twistor string, and a relation to the spinor helicity formalism is also discussed.

1 Introduction and basic results

The Kerr geometry has found application in a very wide range of physical systems: from the rotating black holes and galactic nucleus to fundamental solutions of the low energy string theory. It displays also some remarkable relations to the structure of spinning particles [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

Angular momentum of the spinning particles is very high and the black-hole horizons disappear. Image of the solution changes drastically taking the form of a naked ringlike source. In our old paper [3] this ring was considered as a gravitational waveguide caring the traveling electromagnetic waves which generate the spin and mass of the Kerr spinning particle forming a microgeon with spin. It was conjectured [4] that the Kerr ring represents a closed string, and the traveling waves are the string excitations. It was noted in [6] that in the axidilatonic version of the Kerr solution the field around this ring is similar to the field around a heterotic string, and recently, it was shown that the Kerr ring is a chiral D-string having an orientifold world-sheet [11].

We show here that an analytical twistorial structure of the Kerr spinning particle leads to the appearance of an extra axial stringy system. As a result, the Kerr spinning particle acquires a simple stringy skeleton [12, 13] which is formed by a topological coupling of the Kerr circular string and the axial stringy system, see fig. 1. Meanwhile, there is a third, *complex* string of the Kerr spinning particle which is also connected to the Kerr's holomorphic twistorial structure.

In the very interesting recent paper [14] Witten suggested a string model with a twistorial target space and argued that some remarkable analytical properties of the perturbative scattering amplitudes in Yang-Mills theory [15] have the origin in an holomorphic structure of this string resulting to the holomorphy of the maximally helicity violating (MHV) amplitudes [16, 17]. However, Witten considered this proposal as tentative [14], and other models of strings related to twistors were also suggested. In particular, the original twistor string suggested about fifteen years ago by Nair [16] was recently modified by Berkovits [18] and represented as an alternative to the Witten twistor string. Some other suggestions were also discussed by Siegel [19].

In this connection we pay the attention to the complex Kerr string [7] which plays the key role in the stringy systems of the Kerr geometry and may also represent an alternative to the twistor string.

Contrary to the strings in twistor space, the target space of the complex Kerr string is $\mathbf{CM}^4$ which is the base of a twistor line bundle, so the twistors are adjoined to each point of this string. In many respects this string is similar to the well known $N = 2$ strings. [1]

[1]Note, that there is a cojecture [20] that the Witten twistor string on $\mathbf{CP}^{3|4}$ is indeed equivalent to the $N = 2$ string.

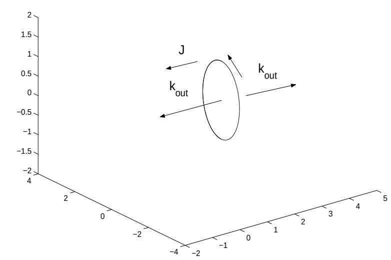

Figure 1: Stringy skeleton of the Kerr spinning particle. Circular D-string and axial stringy system consisting of two semi-infinite D-strings of opposite chiralities.

The complex string appears naturally in the initiated by Newman *complex representation of the Kerr geometry* [7, 8, 10, 23], where the Kerr source is generated by a complex world line $X_0^\mu(\tau)$. Since complex time $\tau = t + i\sigma$ consists of two parameters t and σ, it parametrizes a world sheet, and therefore, the complex world lines can be considered as strings [7, 21]. It turns out that the Kerr complex string is an open string, and its ends are stuck to the two semi-infinite strings of the axial stringy system shown in fig.1. These semi-strings are the lightlike strings of opposite chiralities, and we show that their interplay is described by the massive Dirac equation. So, the axial stringy system may be considered as a stringy carrier of the wavefunction of the Kerr spinning particle.

Therefore, the Kerr spinning particle displays a very interesting stringy system consisting from the Kerr circular string, the axial semi-strings and the complex Kerr string. We show here that these strings have similar analytical properties, and consistency of this system is provided by a common orientifold structure.

Returning to the remarkable simplification of the MHV relativistic scattering amplitudes, one can wonder - which part of the complicated stringy structure of the Kerr spinning particle may be responsible for the very simple holomorphy description of the MHV amplitudes. We discuss it in the end of the paper, arguing that a dominant contribution to the relativistic scattering process may be coming from one of the chiral semi-strings of the Kerr axial stringy system.

In our treatment we prefer to work in the Kerr-Schild formalism [22]

which is based on the Kerr theorem which is related to twistors. For the reader convenience we describe here briefly the necessary details of the real and complex structures of the Kerr geometry. For more details see [7, 8, 23].

2 Twistorial structure of the Kerr congruence

The Kerr-Schild form of the metric is

$$g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is the metric of auxiliary Minkowski space-time (signature $-+++$) in the Cartesian coordinates $x^\mu = (t, x, y, z)$, $h = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}$, and k_μ is a twisting null field which forms the Kerr principal null congruence (PNC) - a family of the geodesic and shear free null curves. The Kerr PNC is shown in Fig. 2. Each null ray of the PNC represents the twistor $Z^a = \{\mu^\alpha, \omega_{\dot\alpha}\}$, in which spinor μ^α determines the null direction, $k_\nu = \bar\mu^{\dot\alpha}\sigma_{\nu\dot\alpha\alpha}\mu^\alpha$, and $\omega_{\dot\alpha} = x^\nu\sigma_{\nu\dot\alpha\alpha}\mu^\alpha$ fixes the position (equation) of the ray.

The Kerr's geodesic and shear-free PNC is determined by *the Kerr theorem* via the solution $Y(x)$ of the algebraic equation $F = 0$, where $F(Y, \lambda_1, \lambda_2)$ is an arbitrary holomorphic function of the projective twistor coordinates $\{Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar\zeta\} = Z^a/Z^0$, and

$$\begin{aligned} 2^{\frac12}\zeta = x + iy, \quad 2^{\frac12}\bar\zeta &= x - iy, \\ 2^{\frac12}u = z - t, \quad 2^{\frac12}v &= z + t \end{aligned} \tag{2}$$

are the null Cartesian coordinates.

In the Kerr-Schild formalism the projective spinor field $Y(x) = \mu^2/\mu^1$ determines the null field $k^\mu(x)$ and other parameters of the solution. In particular, the Kerr-Schild null tetrad is given by

$$\begin{aligned} e^1 &= d\zeta - Y dv, \qquad e^2 = d\bar\zeta - \bar Y dv, \\ e^3 &= du + \bar Y d\zeta + Y d\bar\zeta - Y\bar Y dv, \\ e^4 &= dv + he^3, \end{aligned} \tag{3}$$

and the system of equations $F(Y) = 0; \quad dF(Y)/dY = 0$ determines the singular lines which are caustics of the PNC.

In the Kerr solution function F is quadratic in Y, and the equation $F = 0$ and the system $F(Y) = dF(Y)/dY = 0$ can be explicitly resolved [23] yielding the structure of the Kerr PNC and the Kerr singular ring shown in the Fig.2.

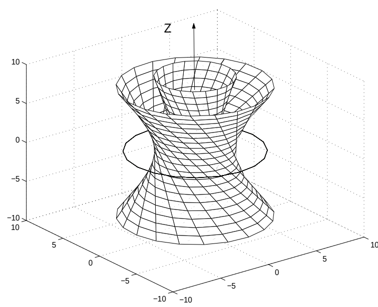

Figure 2: The Kerr circular string and null rays of the PNC which propagate from 'negative' sheet of space-time onto 'positive' one.

The Kerr singular ring is the branch line of the Kerr space on two sheets: 'positive' ($r > 0$) and 'negative' ($r < 0$) ones, so the Kerr PNC propagates from the 'negative' sheet onto 'positive' one through the disk spanned by the Kerr ring. Since the PNC determines a flow of radiation (in radiative solutions), one sees that outgoing radiation is compensated by an ingoing flow on the negative sheet, so the negative sheet acquires the interpretation as a list of advanced fields which are related to the vacuum zero point fields [11, 23, 12]. In this interpretation, the wave excitations can be treated as a result of resonance of a zero point field on the Kerr ring, in the spirit of the semiclassical Casimir effect. It is remarkable, that similar to the quantum case, such excitations of the Kerr string take place without damping since outgoing radiation is compensated by ingoing one. Due to the twofolded topology, the Kerr circular ring turns out to be the "Alice" string, which corresponds to a very minimal nonabelian generalization of the Einstein-Maxwell system. A truncation of the negative sheet leads to the appearance of a source in the form of a relativistically rotating disk [2] and to the class of the superconducting disklike [5] and baglike [9] models of spinning particle.

3 Axial stringy system

Let us consider solutions for traveling waves - electromagnetic excitations of the Kerr circular string. The problem of electromagnetic excitations of the Kerr black hole has been intensively studied as a problem of the quasinormal modes. However, compatibility with the holomorphic structure of the Kerr space-time put an extra demand on the solutions to be aligned to the Kerr PNC, which takes the form $F^{\mu\nu}k_\mu = 0$. The aligned wave solutions for electromagnetic fields on the Kerr-Schild background were obtained in the Kerr-Schild formalism [22]. We describe here only the result referring for details to the papers [12, 13]. Similar to the stationary case [22] the general aligned solution is described by two self-dual tetrad components $\mathcal{F}_{12} = AZ^2$ and $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$, where function A has the form

$$A = \psi(Y, \tau)/P^2, \tag{4}$$

$P = 2^{-1/2}(1+Y\bar{Y})$, and ψ is an arbitrary holomorphic function of τ which is a complex retarded-time parameter. Function $Y(x) = e^{i\phi}\tan\frac{\theta}{2}$ is a projection of sphere on a complex plane. It is singular at $\theta = \pi$, and one sees that such a singularity will be present in any holomorphic function $\psi(Y)$. Therefore, *all the aligned e.m. solutions turn out to be singular at some angular direction θ.* The simplest modes

$$\psi_n = qY^n \exp i\omega_n\tau \equiv q(\tan\frac{\theta}{2})^n \exp i(n\phi + \omega_n\tau) \tag{5}$$

can be numbered by index $n = \pm1, \pm2, ...$, which corresponds to the number of the wave lengths along the Kerr ring. The mode $n = 0$ is the Kerr-Newman stationary field. Near the positive z^+ semi-axis we have $Y \to 0$ and near the negative z^- semi-axis $Y \to \infty$.

Omitting the longitudinal components and the radiation field γ one can obtain [12, 13] the form of the leading wave terms

$$\mathcal{F}|_{wave} = f_R\, d\zeta \wedge du + f_L\, d\bar{\zeta} \wedge dv, \tag{6}$$

where $f_R = (AZ)_{,1}$; $\qquad f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1}$ are the factors describing the "left" and "right" waves propagating along the z^- and z^+ semi-axis correspondingly.

The behavior of function $Z = P/(r + ia\cos\theta)$ determines a singularity of the waves at the Kerr ring, so the singular waves along the ring induce, via function Y, singularities at the $z^{\pm}$ semi-axis. We are interested in the asymptotical properties of these singularities. Near the z^+ axis $|Y| \to 0$, and by $r \to \infty$, we have $Y \simeq e^{i\phi}\frac{\rho}{2r}$ where ρ is the distance from the z^+ axis. Similar, near the z^- axis $Y \simeq e^{i\phi}\frac{2r}{\rho}$ and $|Y| \to \infty$. The parameter $\tau = t - r - ia\cos\theta$ takes near the z-axis the values $\tau_+ = \tau|_{z^+} = t - z - ia, \quad \tau_- = \tau|_{z^-} = t + z + ia$.

For $|n| > 1$ the solutions contain the axial singularities which do not fall of asymptotically, but are increasing that means instability. Therefore, only the wave solutions with $n = \pm 1$ turn out to be admissible. The leading singular wave for $n = 1$,

$$\mathcal{F}_1^- = \frac{4qe^{i2\phi + i\omega_1\tau_-}}{\rho^2}\, d\bar{\zeta} \wedge dv, \tag{7}$$

propagates to $z = -\infty$ along the z^- semi-axis.

The leading wave for $n = -1$,

$$\mathcal{F}_{-1}^+ = -\frac{4qe^{-i2\phi + i\omega_{-1}\tau_+}}{\rho^2}\, d\zeta \wedge du, \tag{8}$$

is singular at z^+ semi-axis and propagates to $z = +\infty$. The described singular waves can also be obtained from the potential $\mathcal{A}^\mu = -\psi(Y,\tau)(Z/P)k^\mu$. The $n = \pm 1$ partial solutions $\mathcal{A}_n^{\pm}$ represent asymptotically the singular plane-fronted e.m. waves propagating along z^+ or z^- semi-axis without damping. The corresponding self-consistent solution of the Einstein-Maxwell field equations are described in [12]. They are singular plane-fronted waves having the Kerr-Schild form of metric (1) with a constant vector k^μ. For example, the wave propagating along the z^+ axis has $k^\mu dx^\mu = -2^{1/2}du$). The Maxwell equations take the form $\Box\mathcal{A} = J = 0$, where Box is a flat D'Alembertian, and can easily be integrated leading to the solutions $\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})]f^+(u)du, \qquad \mathcal{A}^- = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})]f^-(v)dv$, where $\Phi^{\pm}$ are arbitrary analytic functions, and functions $f^{\pm}$ describe the arbitrary retarded and advanced waves. Therefore, the wave excitations of the Kerr ring lead to the appearance of singular pp-waves which propagate outward along the z^+ and/or z^- semi-axis.

These axial singular strings are evidences of the axial stringy currents, which are exhibited explicitly when the singularities are regularized. Generalizing the field model to the Witten field model for the cosmic superconducting strings [24], one can show [13] that these singularities are replaced by the chiral superconducting strings, formed by a condensate of the Higgs field, so the resulting currents on the strings are matched with the external gauge field.

The stringy system containing the chiral modes of only one direction cannot exist since it is degenerated in a word-line [11]. A combination of two $n = \pm 1$ excitations leads to the appearance of two semi-infinite singular D-strings of opposite chirality as it is shown at the fig.1. Similar to the all ray of PNC, the semi-infinite singularities can be extended to the negative sheet passing through the Kerr ring. The world-sheet of such a system acquires the structure of an orientifold and is given by

$$x^\mu(t,z) = \frac{1}{2}[(t-z)k_R^\mu + (t+z)k_L^\mu], \tag{9}$$

where the lightlike vectors k^μ are constant and normalized. At the rest frame the time-like components are equal $k_R^0 = k_L^0 = 1$, and the space-like components are oppositely directed, $k_R^a + k_L^a = 0, \quad a = 1,2,3$. Therefore, $\dot{x}^\mu = (1,0,0,0)$, and $x'^\mu = (0, k^a)$, and the Nambu-Goto string action $S = \alpha'^{-1} \int\int \sqrt{(\dot{x})^2(x')^2 - (\dot{x}x')^2} dt dz$ can be expressed via k_R^μ and k_L^μ.

For the system of two D-strings in the rest one can use the gauge with $\dot{x}^0 = 1, \quad \dot{x}^a = 0$, where the term $(\dot{x}x')^2$ drops out, and the action takes the form $S = \alpha'^{-1} \int dt \int \sqrt{p^a p_a} d\sigma$, where $p^a = \partial_\sigma x^a = \frac{1}{2}[x_R'^\mu(t+\sigma) - x_L'^\mu(t-\sigma)]$.

To normalize the infinite string we have to perform a renormalization putting $m = \alpha'^{-1} \int (x')^2 dz$, which yields the usual action for the center of mass of a pointlike particle $S = m \int \sqrt{(\dot{x})^2} dt$.

4 The Dirac equation

It is known that in the Weyl basis the Dirac current can be represented as a sum of two lightlike components of opposite chirality

$$J_\mu = e(\bar{\Psi}\gamma_\mu \Psi) = e(\chi^+ \sigma_\mu \chi + \phi^+ \bar{\sigma}^\mu \phi), \tag{10}$$

where $\Psi = \begin{pmatrix} \phi_\alpha \\ \chi^{\dot\alpha} \end{pmatrix}$. It allows us to conjecture that the Dirac equation may describe the Kerr axial stringy system - the lightlike currents of two opposite chiralities which are positioned on the different folds of the world-sheet.

In this case the four-component spinor Ψ, satisfying the massive Dirac equation $(\gamma^\mu \hat{P}_\mu - m)\Psi = 0, \qquad \hat{P}_\mu = i\hbar\partial_\mu$, describes an interplay of the axial string currents on the different folds of the Kerr space. In the Weyl basis the Dirac system splits into

$$m\phi_\alpha = i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \chi^{\dot\alpha}, \qquad m\chi^{\dot\alpha} = i\bar\sigma^{\mu\dot\alpha\alpha}\partial_\mu \phi_\alpha. \tag{11}$$

Regarding the function Y as a projective spinor field $Y = \phi_2/\phi_1$, one sees that near the z^+ semi-axis $Y \to 0$, so one can set in this limit $\phi_\alpha = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. This spinor describes the lightlike vector

$$k_R = d(t - z) = \bar\phi_{\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_\mu dx^\mu \phi_\alpha. \tag{12}$$

Similar, near the z^- semi-axis $\bar Y \to \infty$, and this limit corresponds to the spinor $\bar\chi^\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ which describes the lightlike vector

$$k_L = d(t + z) = \bar\chi^\alpha \sigma_{\mu\alpha\dot\alpha} dx^\mu \chi^{\dot\alpha}. \tag{13}$$

The vectors k_L and k_R are the generators of the left and right chiral waves correspondingly. The functions ϕ and χ are fixed up to arbitrary gauge factors. Forming the four component spinor function

$$\Psi = \mathcal{M}(p_\mu x^\mu) \begin{pmatrix} a\phi_\alpha \\ b\chi^{\dot\alpha} \end{pmatrix}, \tag{14}$$

we obtain from the Dirac equations (11)

$am = (p_0 - p_z) b \ln \mathcal{M}', \qquad bm = (p_0 + p_z) a \ln \mathcal{M}',\ p_x + ip_y = 0$, which realizes the Dirac idea on splitting the relation $p_0^2 = m^2 + p_z^2$, and

$$\mathcal{M} = e^{-i\omega t + izp_z}. \tag{15}$$

This solution describes a wavefunction of a free spinning particle moving along the z-axis. It oscillates with the Compton frequency, which is determined by excitations of the Kerr circular string, and leads to a plane fronted

modulation of the axial string by de Broglie periodicity. It allows us to conjecture that the axial stringy system acquires in the Kerr spinning particle the role of *a stringy carrier of wavefunctions*.

5 Orientifold and a complex Kerr string

The tension for a free particle tends to zero, but it can be finite for the bounded states when the axial string forms the closed loops. An extra tension can appear in the bounded systems, since an extra magnetic flow can concentrate on the closed loops. By formation of the closed loops chiral modes have to be matched forming an orientifold structure. The position x^μ of the (say)'left' semi-string, being extended to the 'negative' sheet, has to coincide with the position of the 'right' one. For a free stationary particle it is realized. However, in general case it puts a very strong restriction on the dynamics of the string. The orientifold of the axial string is also suggested by the structure of superconducting strings. As it pointed out by Witten [24], it contains the light-like fermions of both the chiralities, moving in opposite directions, so the massive Dirac solutions for a superconducting string describe an interplay of the trapped fermions of the opposite chiralities. In general case orientifolding the world-sheet requires a very high degree of symmetry for the string. It turns out that this is provided by the holomorphy and orientifold structure of the third, *complex string* of the Kerr geometry [7].

The complex representation of the Kerr geometry [23, 7, 8, 10] is generated by a complex world line $X_0^\mu(\tau) \in \mathbf{CM}^4$, where $\tau = t_0 + i\sigma$ is a complex time parameter. In the stationary Kerr case $X_0^\mu(\tau) = (\tau, 0, 0, ia)$. The complex world line forms a world sheet and takes an intermediate position between particle and string [7, 21]. The real fields on the real space-time x^μ are determined via a retarded-time construction, where the vectors $K^\mu = x^\mu - X_0^\mu(\tau)$ have to satisfy the complex light-cone constraints $K_\mu K^\mu = 0$. Looking on the complex retarded-time equation

$$t - \tau \equiv t - t_0 - i\sigma = \tilde{r}, \tag{16}$$

where $\tilde{r} = r + ia\cos\theta$ is the Kerr complex radial distance (θ is a direction of the null ray (twistor), and r is the spatial distance along the ray), one sees that on the real space-time $\sigma = -a\cos\theta$. Therefore, the light-cone constrains select a strip on τ plane, $\sigma \in [-a, +a]$, and the complex world sheet $X_0^\mu(t, \sigma)$

acquires the boundary, forming *an open complex string* with target space $\mathbf{CM}^4$ which is a base of a twistor bundle. In many aspects this string is similar to the $N=2$ string [21, 26], but it has the signature $(-+++)$ and the euclidean world sheet.

The complex light cones, adjoined to each point of the world sheet, split into the 'right' and 'left' null planes which are the twistors having their 'origins' X_0^ν at the points of the world sheet. The 'left' null planes - twistors $Z^a = \{\mu^\alpha, X_0^\nu \sigma_{\nu\dot{\alpha}\alpha}\mu^\alpha\}$ - form a holomorphic twistor subspace. The two twistors which are joined to the ends of the complex string, $X_0^\nu(t \pm ia)$, have the directions $\theta = 0, \pi$ and are generators of the singular $z^\pm$ semi-strings, so the complex string turns out to be a D-string which stuck to two singular semi-strings of opposite chiralities. Therefore, $z^\pm$ singular strings may carry the Chan-Paton factors which will play the role of quarks with respect to the complex string. Extending this analogy one can assume that the Kerr circular lightlike string, adjoined to the 'center' of the complex string $\sigma = 0$, may carry the factors of the third quark of this stringy system. In the nonabelian generalizations, the chiral singular strings may carry the color currents [16] connected with color traveling waves [25] which play the role of the color quarks.

It was obtained in [7, 11] that boundary conditions of the complex string demand the orientifold structure of the world-sheet. The resulting field equations have the solutions which are satisfied by the holomorphic (!) modes $X_0^\mu(\tau)$. These solutions are the 'left' modes, and the 'right' modes appear by orientifolding. The interval for parameter $\sigma = a\cos\theta$ is doubled: $\Sigma_\pm = [-a, a]$, forming a circle $S^1 = \Sigma_+ \bigcup \Sigma_-$ which is parametrized by $\theta \in [0, 2\pi]$, so Σ_- parametrizes the string in opposite orientation. Orientifold is formed by a Z_2 factorization of the world-sheet. The string turns into a closed but folded one. The 'right' modes have the form of the 'left' ones but have a support on the reversed interval $\theta \in [\pi, 2\pi] \in \Sigma_-$:

$$X_{0R}^\mu(t + ia\cos(2\pi - \theta)) = X_{0L}^\mu(t + ia\cos\theta), \tag{17}$$

which is a well-known kind of extrapolation [26]. So, for the period $(0, 2\pi)$ occurs a flip of the modes $left \rightarrow right$. It is accompanied by a space reversal of the twistors joined to the points of the world-sheet, which is performed by the exchange $Y \rightarrow -1/\bar{Y}$. As a result the orientifold structure of the complex string provides the orientifold structure for the chiral axial strings.

It can be shown that all the three orientifold structures can be joined forming a unite orientifold matched to the holomorphic twistorial structure of the Kerr space-time. We intend to discuss it in details elsewhere.

6 Relation to the spinor helicity formaslism

Let us finally discuss the possible relation to the MHV scattering amplitudes . For the massless and relativistic particles with spin, the external line factors of the amplitudes are described in the spinor helicity formalism [14, 15, 16, 17] which is based on a color decomposition and a reduced description in terms of the lightlike momentum $p_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} p_{\mu}$ and $\pm$ helicities.

The momentum $p_{\alpha\dot{\alpha}}$ is represented via the undoted spinors $< p| = p^{\alpha}$, $|p >= p_{\alpha}$ and doted spinors $[p| = p^{\dot{\alpha}}, \quad |p] = p_{\dot{\alpha}}$ in the form

$$p = \pm |p > [p|. \tag{18}$$

Since the Kerr spinning particle is massive, its momentum along the z-axis is represented as a sum of the lightlike parts $p^{\mu} = p^{\mu}_{L} + p^{\mu}_{R}$, where the corresponding spinors are

$$p^{\alpha}_{L} =< p_L| = p_L \cdot < k_L| \tag{19}$$

and

$$p^{\alpha}_{R} =< p_R| = p_R \cdot < k_R|. \tag{20}$$

For a relativistic motion we have either $p_L << p_R$ or $p_L >> p_R$, which determines the sign of helicity, and as a result one of the axial semi-strings turns out to be strongly dominant for the scattering. It may justify the use of the very reduced description of the Kerr spinning particle via the spinor helicity formalism. We are led to the conclusion that the axial stringy system may be responsible for the high energy scattering processes.

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